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# Enhancement of forced convection heat transfer in viscous fluid flows

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**Abstract**—This paper treats augmentation of convective heat transfer in viscous fluid flows. An analytical investigation of thermal resistance and wall temperature drop in turbulent boundary layers in gases and liquid flows is presented. Various possibilities of augmentation are considered, including artificial disturbance of the boundary layer, artificial flow turbulization, and extended surfaces.

## INTRODUCTION

AUGMENTATION of heat transfer and improvement of power efficiency are the principle goals in the development of heat exchangers. The enhancement of convective heat transfer in single-phase flow of different liquids and low-temperature gases is very important. At present more and more attention is being paid to this problem [1–5]. Here we consider mainly enhancement of forced convection heat transfer in viscous fluid flows.

The amount of heat transfer can be increased by various ways; the first one is a proper selection of working fluids, since $Nu \sim Pr^n$. For example, in laminar flow $Nu \sim Pr^{0.33}$ and in turbulent flow over a plate $Nu \sim Pr^{0.43}$. Then at $Re$ = idem the relationship between $Nu_2$ for transformer oil and $Nu_1$ for air in turbulent flows at $t = 20°C$ could be expressed in the following way:

$$\frac{Nu_2}{Nu_1} = \frac{Pr_2^{0.43}}{Pr_1^{0.43}} = \frac{(370)^{0.43}}{(0.703)^{0.43}} = 14.78. \qquad (1)$$

So the heat transfer in liquids flows is several times higher than in air flow.

The heat transfer rate is mainly governed by the flow velocity and the flow pattern. Normally a higher flow velocity implies a higher heat transfer coefficient. At the same time, high coolant velocities are obtained at the penalty of increasing pumping power. Depending on velocity and Reynolds number there are laminar or turbulent flows.

A turbulent flow means a higher effective viscosity with a considerably higher momentum flux in comparison with laminar flow. This difference is reflected in the heat transfer rate. In a laminar boundary layer over a plate the heat transfer coefficient $\alpha \sim u^{0.5}$ and in a turbulent boundary layer $\alpha \sim u^{0.8}$ where $u$-flow velocity. So from the point of augmentation the turbulent flow is more advantageous.

In an interaction between a heat transfer surface and a viscous flow, the main thermal resistance is exerted by the boundary layer which develops on the surface. The larger its thickness and the lower the conductivity, the lower the heat transfer rate. Therefore any attempt to enhance convective heat transfer must involve a reduction of thermal resistance in the boundary layer. Two methods of augmentation are most effective: artificial disturbance of the boundary layer and artificial flow turbulization.

Enhancement of convective heat transfer can be obtained by passive or active methods. Passive methods are mainly directed to free stream turbulization, disturbance of the boundary layer or otherwise. Active or external methods, such as vibrating or rotating the hot surface, using an ultrasonic or electric field, require additional external energy. These methods are effective in gases and laminar flows, but they are negligible in fluids. So in this paper we will deal with passive methods.

Common opinion on the effectiveness of the augmentation of heat transfer in fluid of different $Pr$ could not be achieved for quite a long time. Popular analysis by Nunner [6] suggested that at high $Pr$ the influence of surface roughness on heat transfer was very small and the usefulness of this method was confined only to gas flows; Kestin [7] and others studied the effect of free-stream turbulence on heat transfer rates mainly in gas flows. So the opinion prevailed that the augmentation of the heat transfer is not effective in viscous fluid flows.

Both analytical study of the turbulent boundary layer and the heat transfer in the boundary layer and our measurements suggested that the heat transfer augmentation in viscous fluid flows could also be effective. In this case we should pay more attention to the disturbance of the viscous sublayer.

## THERMAL RESISTANCE OF A TURBULENT BOUNDARY LAYER

As already noted, the main thermal resistance in a solid–fluid interaction comes through the formation of a boundary layer, and efforts towards enhancing heat transfer must be directed at artificially destroying or disturbing the boundary layer. In this respect, the most advantageous case is that of a turbulent boundary layer.

## NOMENCLATURE

| | |
|---|---|
| $c_p$ | heat capacity [J kg$^{-1}$ °C$^{-1}$] |
| $f$ | hydraulic drag factor |
| $k$ | height of roughness elements [mm] |
| $k^+$ | dimensionless roughness height, $ku_*/v$ |
| $l$ | mixing length of turbulence [m] |
| $Nu$ | Nusselt number, $\alpha d/\lambda$ |
| $Pr$ | Prandtl number, $v/a$ |
| $Pr_t$ | turbulent Prandtl number, defined by equation (6) |
| $q$ | heat flux [W m$^{-2}$] |
| $Re$ | Reynolds number, $Ud/v$ |
| $St$ | Stanton number, $\alpha/(\rho u c_p)$ |
| $t$, $T$ | temperature [°C, K] |
| $Tu$ | turbulence intensity, $\sqrt{(u'^2/U)}$ |
| $u$, $v$ | velocity components [m s$^{-1}$] |
| $u'$, $v'$ | instantaneous velocity components [m s$^{-1}$] |
| $u_*$ | friction velocity $\sqrt{(\tau_w/\rho)}$ [m s$^{-1}$] |
| $u^+$ | dimensionless velocity, $u/u_*$ |
| $U$ | free stream velocity [m s$^{-1}$] |
| $y^+$ | dimensionless distance, $yu_*/v$. |

Greek symbols

| | |
|---|---|
| $\alpha$ | heat transfer coefficient $k$ [W m$^{-2}$ K$^{-1}$] |
| $\delta^{**}$ | momentum thickness [m] |
| $\varepsilon_\tau$ | eddy viscosity coefficient [m$^2$ s$^{-1}$] |
| $\varepsilon_q$ | eddy thermal diffusivity coefficient [m$^2$ s$^{-1}$] |
| $\theta$ | temperature measured from the wall temperature [°C, K] |
| $\theta_*$ | reference temperature, $q_w/\rho c_p u_*$ [K] |
| $\theta^+$ | dimensionless temperature, $\theta/\theta_*$ |
| $\lambda$ | thermal conductivity [W m$^{-1}$ K$^{-1}$] |
| $\mu$ | dynamic viscosity [N s m$^{-2}$] |
| $v$ | kinematic viscosity, $\mu/\rho$ [m$^2$ s$^{-1}$] |
| $\rho$ | density [kg m$^{-3}$] |
| $\tau$ | shear stress [N m$^{-2}$] |
| $\phi$ | angle [deg., rad]. |

A turbulent boundary layer flow near a plate can be subdivided into two regions: the wall, or inner part of the boundary layer, and its outer part. The wall region occupies about 20% of the thickness of the entire boundary layer and the flow in it is described by the law of the wall. Within the wall region, adjacent to the wall, at $y^+ < 7$, exists a very thin viscous sublayer, where the flow is controlled by molecular viscosity. This sublayer is followed by a transition zone where in spite of significant turbulent fluctuations, the effect of viscosity must be considered. Then at $y^+ > 25$ the flow acquires a developed turbulent nature and the velocity distribution becomes logarithmic. The flow in the wall region is described by the law of the wall $u^+ = f(y^+)$.

The outer part of the boundary layer, which extends over 80% of the boundary layer thickness, obeys the law of the wake. Similar to the formation of the velocity layer, a thermal boundary layer forms over a surface. The heat flux that overcomes the resistance of the viscous sublayer propagates over the entire turbulent boundary layer and the thicknesses of the thermal and velocity boundary layers coincide.

The main difficulty in evaluating the heat transfer is presented by the molecular and turbulent transfer contributions with the growth of $Pr$. Measurements of velocity profiles across the turbulent boundary layer of a plate in different fluids revealed the universal nature of the wall law, including the logarithmic equation which is valid for different fluids. For the wall layer we have the following equations:

$$q_w = \lambda \frac{d\theta}{dy} - \rho c_p \overline{v'\theta'} \tag{2}$$

$$\tau_w = -\mu \frac{du}{dy} - \rho \overline{u'v'}. \tag{3}$$

Hence,

$$-\rho \overline{u'v'} = \rho l^2 \left(\frac{du}{dy}\right)^2 \tag{4}$$

$$\varepsilon_\tau = l^2 \frac{du}{dy} \tag{5}$$

$$Pr_t = \frac{\overline{u'v'}}{\overline{v'\theta'}} \frac{d\theta/dy}{du/dy}. \tag{6}$$

Then

$$-\overline{\theta'v'} = \frac{l^2}{Pr_t} \frac{du}{dy} \frac{d\theta}{dy} \tag{7}$$

$$q_w = \lambda \frac{d\theta}{dy} + \frac{\varepsilon_\tau}{Pr_t} \frac{d\theta}{dy} \rho c_p. \tag{8}$$

Using reference temperature

$$\theta_* = q/\rho c_p u_* \tag{9}$$

for the wall layers, proceeding from the heat flux equation (8). Vaitiekūnas *et al.* [8] obtained the following equation:

$$\left(\frac{1}{Pr} + \frac{1}{Pr_t} \frac{\varepsilon_\tau}{v}\right) \frac{d\theta^+}{dy^+} = 1, \tag{10}$$

where $Pr_t = \varepsilon_\tau/\varepsilon_q$ is the turbulent Prandtl number, $\varepsilon_\tau$ and $\varepsilon_q$ are the coefficients of eddy viscosity and heat conductivity, $v$ is kinematic viscosity; $\theta^+ = \theta/\theta_*$, $\theta_* = q_w/(qc_p u_*)$, $u_* = \sqrt{\tau/\rho}$, $\theta$ is temperature, $q$ is

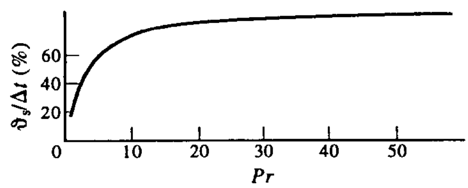


FIG. 1. Temperature drop in the viscous sublayer depending on $Pr$ number.

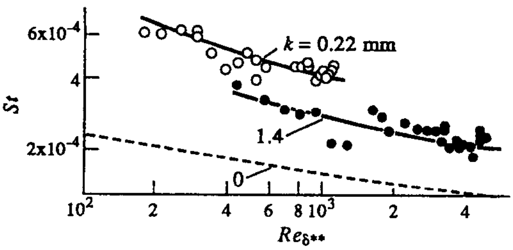


FIG. 2. Heat transfer from a rough-surface plate in a flow of transformer oil.

heat flux, $c_p$ is specific heat capacity; $\tau$ is shear stress, $\rho$ is density.

From equation (10) we see that at high $Pr$ the relative contribution of turbulent transfer increases: at a fixed $Re$ and increasing $Pr$, the first term on the left-hand-side goes to zero and the second term, which is small for the viscous sublayer, may become much larger than the term for molecular transfer. Thus with an increase of $Pr$ the viscous sublayer shows an increasing resistance to heat transfer.

In Fig. 1 analysis of the relation between the temperature gradient in the viscous sublayer and the value of $Pr$ shows that the viscous sublayer is responsible for 25% of the total fluid-to-wall temperature difference in air, but 90% in transformer oil with $Pr = 55$.

## DISTURBANCE OF THE BOUNDARY LAYER BY ROUGHNESS ELEMENTS

Nunner [6] maintained that surface roughness generated vortices mainly on the boundary of the turbulent core, which had no effect on the viscous sublayer and this method for augmentation of high $Pr$ numbers fluids is not effective. Our analysis shows that in air the main contribution to thermal resistance is in the turbulent boundary layer, but in liquids with $Pr \gg 1$, it is mainly in the viscous sublayer. Therefore, heat transfer should be enhanced by the action of surface roughness in the wall layers. We encounter a new problem here, that of choosing the suitable height, shape and location of the surface roughness elements for each fluid in relation to its type and physical properties.

Thus, in order to reach the necessary heat transfer enhancement in gases, the whole wall layer must be disturbed. But in liquids with high $Pr$ only the viscous sublayer should be disturbed. The higher the $Pr$ number, the shorter are the surface elements needed for a significant heat transfer enhancement. One has to bear in mind that in liquids it is a transition region of roughness that gives most intensive heat transfer rates. In air, with a growth of height $k$ of roughness elements, a continuous increase of the heat transfer is observed. In transformer oil, according to the measurements by Drižius *et al.* [9], a maximum increase of the heat transfer coefficient, as described by the Stanton number, is observed at a roughness $k = 0.22$ mm. A further increase of roughness up to $k = 1.4$ mm results in a decrease of $St$ (Fig. 2). In liquids it is a transition region of roughness that gives most intensive heat transfer rates. The dimensionless height $k^+$ is introduced to describe the roughness regime:

$$k^+ = \frac{ku_*}{\nu}, \tag{11}$$

where $k$ is height of roughness.

Measurements suggest that $k^+ < 5$ implies surface elements submerged in the viscous sublayer and having no influence on the heat transfer rate. At $k^+ > 70$ a full roughness effect is observed. The range of $k^+$ from 5 to 70 covers transitional region of its effect.

Heat transfer curves obtained with rough-surface plates suggest different influence of $k^+$ on heat transfer with a variation of $Pr$. In air a continuous increase of the heat transfer is observed with a growth of $k^+$ (Fig. 3). In transformer oil a maximum increase of the heat transfer coefficient, as described by the Stanton number, is observed at a transition region roughness effect at $k^+ < 70$. A further increase of $k^+$ results in a decrease of $St$. In water at $Pr = 5.4$ a maximum increase of the heat transfer coefficient, as described by the Stanton number, is also observed at a transition region of roughness at $k^+ < 70$. A further increase of $k^+$ results in a decrease of $St$ [9].

Therefore, measurements confirm the suggestion that in viscous liquid flows the heat transfer aug-

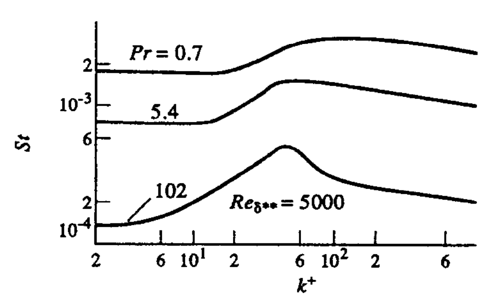


FIG. 3. Heat transfer from a rough surface at different $Pr$. $\delta^{**}$-thickness of the momentum loss.

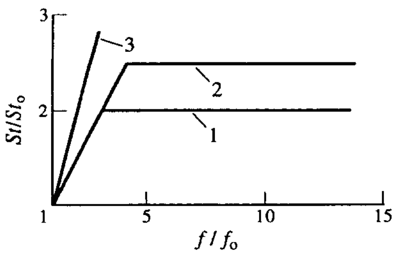


FIG. 4. Heat transfer in a flat channel with two rough opposite walls [10]: 1, sand-type roughness; 2, transverse ribs; 3, line of constant increase of $St/St_0 = f/f_0$.

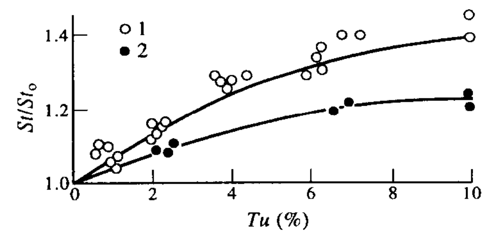


FIG. 5. The effect of free stream turbulence $Tu$ on the heat transfer for a turbulent boundary layer on a plate: 1, $Pr = 0.7$; 2, $Pr = 5.4$.

mentation by roughness elements is also possible, but in this case low roughness elements should be used.

The process of heat transfer enhancement inside rough-surface pipes, especially in the entrance region, is similar to the above-mentioned case of rough-surface plates.

General tendencies of the heat transfer augmentation techniques by rough surfaces are surveyed in Fig. 4. Here for $f/f_0$ up to 3 or 4 the heat transfer augmentation due to the surface roughness is nearly proportional to the increase of hydraulic drag [10]. Transverse ribs are more effective then sand-type roughness. This supports the speculation over regular separation.

The experiments suggest that on specific surfaces the heat transfer augmentation can even exceed the increase of hydraulic drag. It usually occurs when the flow is influenced by tender three-dimensional disturbances of variable pressure gradients.

## FREE STREAM TURBULIZATION

Artificial flow turbulization is one of the effective means of heat transfer enhancement. Main flow turbulization has a significant effect on heat transfer in a laminar boundary layer. It has been established that an increased flow turbulence exerts a strong effect on the hydrodynamics and heat transfer even in the case of a developed turbulent flow in the boundary layer on the plate. An analysis shows that this influence is exerted through the disturbance of the outer zone of the boundary layer while the wall region remains unaffected. But, as mentioned, with an increase of $Pr$ the viscous sublayer shows main resistance to heat transfer. So increased flow turbulence should exert a strong effect on heat transfer in the case of air flow, but there should be less effect for viscous liquids, since the wall region of the turbulent boundary layer is very conservative with respect to external effects. This was supported by experiments on the plate by Šlančiauskas and Pedišius [11]. They show (Fig. 5) that an increased flow turbulence exerts a strong effect on the heat transfer ($St$) in the case of a developed air flow in the turbulent boundary layer. But, as we see from Fig. 5 this effect in water flows is less. In Fig. 5 $St_0$ is the Stanton number for non-turbulent flow.

Analysis of these data shows that free stream turbulization influence is exerted through the disturbance of the outer zone of turbulent boundary layer while the wall region remains unaffected.

## ENHANCEMENT OF HEAT TRANSFER FROM TUBES IN CROSS-FLOW

Tubes constitute important elements of heat transfer surfaces. Therefore great attention is paid to the enhancement of heat transfer in the case of flow over tubes. At low Reynolds numbers the heat transfer at the front part of the tube is maximum and it decreases downstream with the growth of the laminar boundary layer thickness on the tube. In the region downstream of the separation of the laminar boundary layer at 84° the heat transfer gradually increases (curve at $Tu = 1\%$ in Fig. 6).

A high free stream turbulence aids the formation of the turbulent boundary layer on the tube surface, thus enhancing the heat transfer. As shown in Fig. 6, at $Re = 6.8 \times 10^4$, in the flow of water, an increase of turbulence intensity from 1 to 6.8% is not only accompanied by an increase of the heat transfer at the

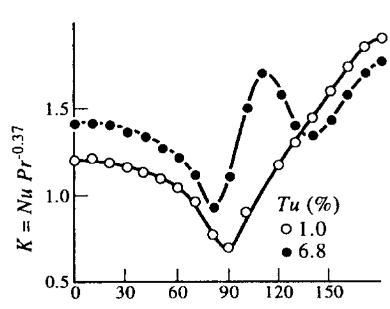


FIG. 6. The effect of free stream turbulence on the local heat transfer from a cylinder.

front part of the cylinder, but the laminar boundary layer, instead of separating at $\phi = 90°$, becomes turbulent and separates at $\phi = 140°$; this also adds to the heat transfer augmentation [12].

Our measurements [12] show that with the increase of free stream turbulence from 1.2% to 15%, the average heat transfer from tube in cross-flow increases about 40% in the subcritical flow regime ($Re < 2\times10^5$), and about 55% in the supercritical flow regime ($Re > 2\times10^5$).

The behaviour of heat transfer is also similar in external flows over rough-surface tubes. Suitable surface elements may provide significant heat transfer enhancement from tubes in cross-flow.

A two- and three-fold increase of local heat transfer is observed on rough surfaces, as compared to smooth ones; Fig. 7. The presence of roughness introduced a sharp change of the hydrodynamic pattern over the surface. The average heat transfer [1] suggests that the onset of the critical flow regime is strongly dependent on surface roughness, and occurs at lower $Re$ values and at larger roughness heights. At low $Re$ the laminar boundary layer covers surface elements completely and the average heat transfer is not influenced by surface roughness. In the higher range of $Pr$ and $Re$ the boundary layer thickness is considerably smaller than the height of roughness elements, and average heat transfer is again independent of surface roughness.

An optimal surface roughness must be found for each specific case and a given flow regime. Figure 8 illustrates a 40% increase of the average heat transfer due to rough-surface tube bundles, compared to smooth ones [13].

Successful heat transfer augmentation by two-dimensional surface roughness has been observed. A two-dimensional surface roughness on tubes in transverse bundles [13] at $Re < 2\times10^5$ resulted in a higher power efficiency, combined with a heat transfer augmentation similar to the case of a three-dimensional surface roughness.

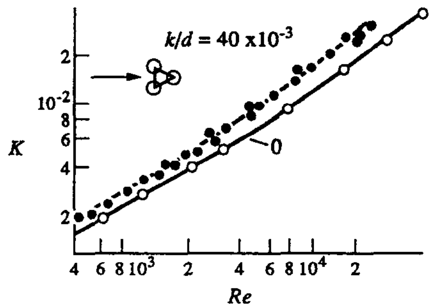


Fig. 8. Mean heat transfer of a rough-surface cylinder in a $1.25\times0.935$ bundle in transformer oil flow [13]. $K = Nu\cdot Pr^{-0.4}$.

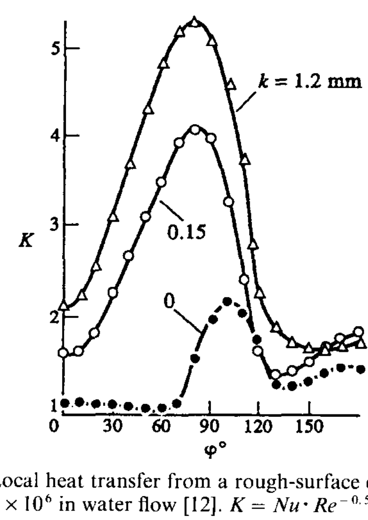


Fig. 7. Local heat transfer from a rough-surface cylinder at $Re = 1\times10^6$ in water flow [12]. $K = Nu\cdot Re^{-0.5}\cdot Pr^{-0.4}$.

## EXTENDED SURFACES

The amount of heat transfer from a surface to a flow at a constant temperature difference is a function of the surface area and of the heat transfer coefficient. Quite often, when two fluids are involved, and one (say air) has a considerably lower heat transfer coefficient than the other (say water), the previously discussed methods of heat transfer enhancement are ineffective. Then heat transfer may be significantly enhanced by the use of extended surfaces, mostly by applying finned tubes. This is true in spite of the fact that the heat transfer coefficient of a finned tube is usually lower than that of a smooth tube.

Effective values of fin pitches and fin heights are determined by the boundary layer thickness. The actual height of fins must exceed the boundary layer thickness to extend the active heat transfer surface. Finned surfaces are most efficient in gaseous flows, but their application spreads also to liquid flows. Construction of specific configurations for liquid flows constitutes one of the problems for researchers and designers. In viscous liquids the thermal boundary layers are relatively thin, so the fins must be lower than in gaseous flows; where the thermal boundary layer is relatively thick, fins as high as 25 mm are necessary, but in liquids they are up to 1.5 mm high. In gas and liquids flows smooth tube bundles have higher heat transfer coefficients than finned ones. But bundles of finned tubes are more compact, therefore the relative heat transfer of finned tubes is higher than that of smooth ones.

## CONCLUSIONS

The possibilities of the convective heat transfer augmentation in the turbulent boundary layer were considered. Different approaches should be used for liquid and gas flows, because for gases the temperature drop is across the turbulent boundary layer, but in liquids flows it is in the viscous sublayer. Thus in gases all turbulent boundary layers must be disturbed by roughness elements, but in liquids mainly the viscous sublayer should be disturbed. An increased flow turbulence exerts a strong effect on heat transfer in the case of air flow, but less effect in viscous liquids flow.

In extended surfaces for gaseous flows, where the thermal boundary layer is relatively thick, much higher fins are necessary, but in liquid flows they are much lower. In viscous liquid flows augmentation of heat transfer is also effective, as in air flow, but in liquids the main thermal resistance is in the viscous sublayer and this factor should be considered.

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